September 24: Week 4 Problems

Problem 1

A bunch of children are playing in groups on a playground (a group might have only one child in it). Every minute, one child leaves their current group and joins a group that has at least as many children as their previous group. Prove that eventually all of the children are playing in one huge group.

Problem 2 (Putnam 2010)

Given a positive integer n, what is the largest k such that the numbers $1, 2, ..., n$ can be put into k boxes so that the sum of the numbers in each box is the same? [When $n = 8$, the example $\{1, 2, 3, 6\}, \{4, 8\}, \{5, 7\}$ shows that the largest k is at least 3.]

Problem 3 (Putnam 2008)

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function such that $f(x, y) + f(y, z) + f(z, x) = 0$ for all real numbers x, y, and z. Prove that there exists a function $g : \mathbb{R} \to \mathbb{R}$ such that $f(x, y) = g(x) - g(y)$ for all real numbers x and y.

Problem 4

[1] Let $\triangle ABC$ be an equilateral triangle with sides of length 1. Define S to be the set of all points D such that exactly one of the angles $\angle ADB$, $\angle ADC$, and $\angle BDC$ is obtuse. Compute the area of S.

[1] Problems 1 and 4 are from Ronnie Pavlov's Putnam Class at the University of Denver